Reluctance Network Treatment of Skin and Proximity Effects in Multi-conductor Transmission Lines

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Abstract — A simple and flexible method is described to determine the frequency dependent inductance and resistance matrices of multi-conductor transmission lines. The method is based on the reluctances of flux channels between the electrical conductors, determined from their geometry and material parameters. The method is shown to be efficient and reliable at all frequencies. It becomes exact in the limit where both the skin depth and the typical gap width between conductors are much smaller than their thickness, a limit which is particularly hard in conventional FEM calculations. Applications include cross-talk phenomena in signal interconnects and high-frequency modeling of transformer and machine windings.

I. INTRODUCTION

A basic problem in multi-conductor transmission line (MTL) theory [1] is the calculation of per-length parameter matrices **L**, **R** of self and mutual inductances and of resistances, respectively. It is particularly difficult in situations where the conductors are very close to each other, i.e., where the widths of the gaps between them typically are smaller than their diameters. Examples are tightly wound coils like the windings of transformers [2] or electrical machines [3], and interconnects consisting of closely packed wires [4]. In such cases the proximity effect leads to non-negligible off-diagonal elements of **R**, and both **L** and **R** become strongly frequency dependent.

II. RELUCTANCE NETWORK DESCRIPTION OF A MTL

To study these effects, we consider an arrangement of n parallel conductors of unit length, labeled by $k = 1,\dots,n$ (Fig. 1), and surrounded by a shield which is grounded at both ends and thick enough so that the region outside of it is field free at all frequencies of interest.

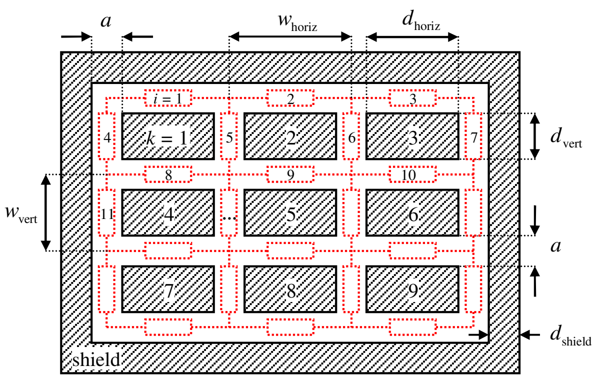

Fig. 1. Cross section of conductor arrangement (not in scale). Conductors and shield are shaded, the reluctance network is shown dotted and in red.

Although our example with evenly spaced, rectangular conductors of equal size is quite symmetric and regular, the method works just as well for any irregular geometry. Also shown in Fig. 1 is the reluctance network representing the available magnetic flux paths [5], so that each network mesh contains exactly one of the conductors. The reluctances are denoted by $\mathfrak{R}_i$ $(i = 1,\dots,s)$, where s is the total number of branches in the network. We can then write the total per-length impedance matrix of the conductor arrangement as

$$\mathbf{Z} = \mathbf{R} + \mathrm{j}\omega\mathbf{L} = \mathrm{j}\omega\left(\boldsymbol{\partial}^{\mathrm{T}}\boldsymbol{\mathfrak{R}}\,\boldsymbol{\partial}\right)^{-1}, \qquad (1)$$

where $\boldsymbol{\partial}$ denotes the $s\times n$ connectivity matrix of the reluctance network (consisting of elements 0 and ±1), and $\boldsymbol{\mathfrak{R}}$ is a $s\times s$ diagonal matrix whose elements are the reluctances for unit conductor length

$$\mathfrak{R}_i = w_i\mu_0^{-1}\left(a_i\mu_{\mathrm{g},i} + \frac{\mu_{i+}\theta_{i+}}{\kappa_{i+}} + \frac{\mu_{i-}\theta_{i-}}{\kappa_{i-}}\right)^{-1}, \qquad (2)$$

with $$\kappa_{i\pm} = (1+\mathrm{j})/\delta_{i\pm}\ , \quad \delta_{i\pm} = \sqrt{2/(\omega\mu_{i\pm}\mu_0\sigma_{i\pm})}\ , \qquad (3)$$

and $$\theta_{i\pm} = \tanh(\kappa_{i\pm}d_{i\pm}/2)\ . \qquad (4)$$

Here w_i is the length of a given gap i in the flux direction and a_i its width. The two adjacent conductors/shield are labeled $i+$ and $i-$ (instead of k) here, see Fig. 2. $\sigma_{i\pm}$ are their conductivities, $\mu_{i\pm}$ their relative permeabilities, and $d_{i\pm}$ their thicknesses in direction perpendicular to the gap; for instance, all horizontal gaps in Fig. 1 have lengths $w_i = w_{\mathrm{horiz}}$ and $d_{i\pm} = d_{\mathrm{vert/shield}}$. $\delta_{i\pm}$ are skin depths at the frequency ω, and $\mu_{\mathrm{g},i}$ refers to the insulating medium in the gap. In principle, all these parameters may have different values for every gap i, although in our example of Fig. 1 they have not.

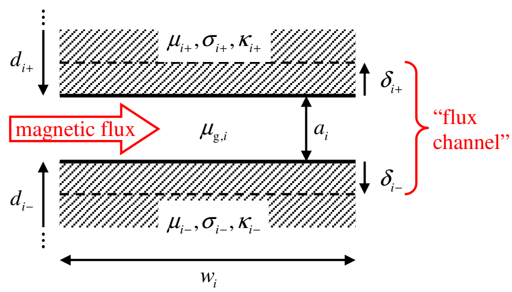

Fig. 2. Local geometry of the gap between two conductors/shield.

In our example we used the values $d_{\mathrm{horiz}} = 7.3$ mm, $d_{\mathrm{vert}} = 3.3$ mm, $\mu_{i\pm} = 1$, $\sigma_{i\pm} = 6\times10^7$ S/m for all conductors; $a_i = a = 0.2$ mm, $w_{\mathrm{horiz/vert}} = d_{\mathrm{horiz/vert}} + a$, $\mu_{\mathrm{g},i} = 1$ for all gaps; and $\mu_{i\pm} = 1000$, $\sigma_{i\pm} = 10^7$ S/m for the shield, which moreover is thick enough so that no flux leaks to the outside (i.e., $\delta_{\text{shield}} < d_{\text{shield}}$) for frequencies between 1 Hz and 10 MHz. These values were chosen to represent copper conductors with an iron shield/armor, although the actual numbers are unimportant here.

The reluctance network method is very flexible and numerically efficient since it is based on the simple analytical formulas (1)−(4). In the full paper we will provide a derivation, and also add to expression (1) a simple correction term for the internal self inductances of the conductors. The reluctance formula (2) is valid for a gap of constant width a_i; more general cases will be discussed in the full paper.

III. COMPARISON TO FEM SIMULATIONS

We have verified the predictions of the reluctance network method by comparison with finite-element field computations, using the commercial FEM tool COMSOL. The results are shown in Figs. 3 and 4 below.

Fig. 3a shows the frequency dependent self inductance L_{11} of conductor 1 including skin and proximity effects due to the other conductors and the shield. Fig. 3b looks at the mutual inductance L_{14} between conductors 1 and 4, which due to the strong magnetic coupling between the conductors is almost equal to L_{11}; therefore the difference $L_{11} - L_{14}$ is shown instead. It is constant below a few 100 Hz where the magnetic flux completely penetrates the conductors ($\delta_{i\pm} > d_{i\pm}$ for all conductors), and decreases towards higher frequencies as the flux is expelled due to the skin effect.

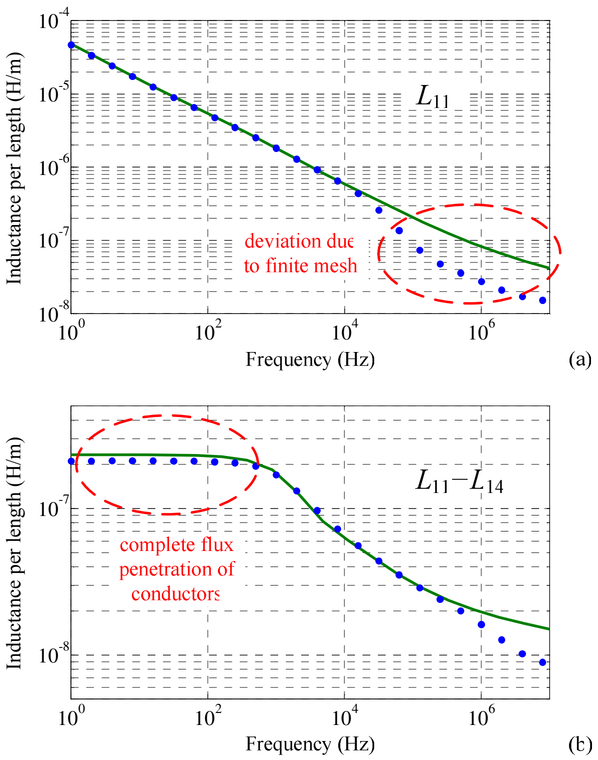

Fig. 3. Self and mutual inductances as functions of frequency, calculated with reluctance network method (line) and FEM (dots).

L_{11} is still increasing towards the lowest frequencies, since even at 1 Hz the flux does not yet completely penetrate the shield ($\delta_{\text{shield}} < d_{\text{shield}}$). The deviations at high frequencies are caused by the finite resolution of the FEM mesh; they move upwards in frequency when the mesh is made finer. This implies that the reluctance network result has the correct behavior at high frequencies.

Fig. 4a shows the resistance R_{11} of conductor 1 including skin and proximity effects. At low frequencies it is approaching the dc value of about 0.7 mΩ/m. Fig. 4b shows the mutual resistance R_{14} between conductors 1 and 4, which approaches zero at low frequencies. At high frequencies, it becomes almost equal to R_{11}, since here the conductor voltages are dominated by the flux in the shield, which is affecting all conductors equally.

In summary, for calculating **L** and **R** matrices of a MTL, the reluctance network method is in very good agreement with FEM down to the lowest frequencies, and it is even superior to it at high frequencies.

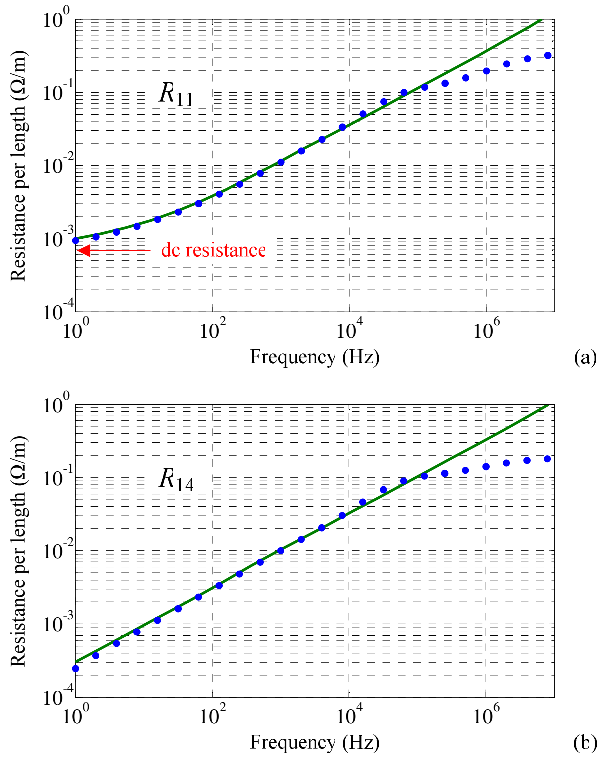

Fig. 4. Self and mutual resistances as functions of frequency, calculated with reluctance network method (line) and FEM (dots).

IV. REFERENCES

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